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# Nonlinear aspects of boiling systems and a new method for predicting the pool nucleate boiling heat transfer

L.H. Chai, X.F. Peng\*, B.X. Wang

Thermal Engineering Department, Tsinghua University, Beijing 100084, China Received 5 November 1998; received in revised form 28 March 1999

## Abstract

Available empirical or phenomenological correlations and mechanistic models developed using average and linear procedures are applicable within relatively narrow ranges and/or for special conditions. The present investigation considers the interactions among active sites or bubbles and the nonlinear dynamic characteristics of the boiling processes to analyze the stochastic and nonlinear features of boiling systems. A bubble size distribution function was derived using a bubble number balance. A new method was then proposed for predicting the pool nucleate boiling heat transfer flux. The present model is in good agreement with previous experimental results, which indicate that the theory presented is more reasonable than traditional theories. © 1999 Elsevier Science Ltd. All rights reserved.

#### 1. Introduction

Boiling processes are encountered in a large number of applications, including not only traditional industrial applications such as metallurgical quenching processes, flooded tube and shell evaporators, and immersion cooling of industrial components, but also modern heat transfer technologies related to space thermal control, electronics components, nuclear reactor, cooling devices, etc. A substantial amount of effort has been devoted to understanding and modeling of the transport phenomena during the boiling process. A plethora of empirical correlations are now available in the literature. However, most of these empirical correlations can only be applied in relatively narrow ranges and/or for special conditions with a considerable error band. Some current correlations even contain different independent variables for boiling phenomena for the same experimental conditions. These conflicts can be

\* Corresponding author. Fax: +86-10-6277-0209. *E-mail address:* pxf-dte@tsinghua.edu.cn (X.F. Peng) attributed to the extreme complexity of boiling systems. At present, it is more difficult to suggest a unique way to predict boiling heat transfer coefficients than for conduction, convection or radiative heat transfer. A complete theory has not been developed because the physical phenomena are not sufficiently well understood. The present investigation focuses on understanding the boiling mechanisms.

The high heat flux in nucleate boiling is commonly attributed to the following three mechanisms [1]:

- 1. Latent heat transfer associated with phase change. As a bubble grows on the surface, a thin liquid layer forms underneath the bubble. The bubble grows by evaporation of the liquid layer.
- 2. Transient conduction and micro-convection heat transfer. This mechanism occurs when the liquid contacts the heating surface after bubble department.
- 3. Natural convection heat transfer. Natural convection heat transfer occurs in regions not occupied by bubbles, but is often enhanced by departing bubbles.

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| ĉ                | constants                           | ν          | parameter defined in Eq. (30)            |
|------------------|-------------------------------------|------------|--|
| $C_{\rm p}$      | specific heat at constant pressure  | $\delta$   | Dirac delta function                     |
| $f^{\mathbf{P}}$ | bubble departure frequency          | $\theta$   | parameter defined in Eq. (29)            |
| F                | fluctuation growth rate             | λ          | thermal conductivity                     |
| G                | Gibbs function                      | ξ          | dimensionless number defined in Eq. (15) |
| $h_{\rm fg}$     | latent heat                         | v          | viscosity                                |
| k                | drift growth rate                   | $\rho$     | density                                  |
| N                | bubble number distribution function | $\sigma$   | surface tension.                         |
| р                | pressure                            |            |  |
| Р                | bubble probability function         | Subscripts |  |
| q                | nucleation rate                     | ave        | average                                  |
| q''              | heat flux                           | с          | critical, convection                     |
| r                | bubble radius                       | e          | equilibrium, evaporation                 |
| $R_{\rm a}$      | roughness                           | 1          | liquid                                   |
| t                | time                                | S          | saturated state                          |
| Т                | temperature.                        | tot        | total                                    |
|                  |                                     | v          | vapor                                    |
| Greek symbols    |                                     | W          | wall.                                    |
| α                | heat transfer coefficient           |            |  |
|                  |                                     |            |  |

These mechanisms have been used to propose many different empirical correlations [1]. Recently developed models in the literature all include these three heat transfer mechanisms [2,3]. The contribution of evaporative heat transfer is a predominant factor. Benjamin and Balakrishnan [2] demonstrated that the contribution of micro-layer evaporation to the heat flux is as high as 40-50% at low to moderate heat fluxes. Latent heat transfer will play an even more important role in high heat flux [4]. Stephan [5] also stated that latent heat transfer is very important to the total heat transfer. The well-known Mikic and Rohsenow model [6] is in agreement with experimental results only when accounting for the contribution of latent heat transfer [7]. However, a salient feature for these theories is that, the heat transported, q''(r), from the surface by the bubbles departing from a single nucleation site is calculated first. Then the nucleate boiling heat transfer is found from the product of q''(r), the bubble number density, N, and the bubble departure frequency, f. Therefore, classical theories depend mainly upon macro-scale phenomena, the mechanisms of boiling heat transfer were found by experimental observations and idealized approximations. Numerous heat transfer correlations have been developed over the decades by modeling the nucleate boiling heat transfer rate using a linearized approach. For example, physical phenomena are analyzed on the basis of a single site or stem, then by uniform conditions are assumed to obtain results for a given active site density distribution. Consequently, important interactions between active sites or bubbles are ignored, which results in a doubtful mechanistic model [8].

The literature survey suggests that the traditional boiling heat transfer paradigm be built upon linear approximations and deterministic bubbles behavior models. The classical theories are therefore unable to get further insight into the nature of and to precisely predict boiling heat transfer in practical applications. This paper accounts for the nonlinear interaction effects and includes a non-uniform bubble size distribution. A stochastic or non-equilibrium statistical theory of boiling is proposed considering bubble growth and active site nucleation. The boiling process is then described by a few basic equations and physical parameters in a unified fashion. The suggested model considers boiling as a transient process, i.e., some embryo bubbles formed to a certain size and continue to grow, some bubble being already departed, while some other embryo bubbles being just formed. Therefore, the bubble size would be stochastic and could not be uniformly distributed on the surface. A new method for calculating the nucleate boiling heat transfer has been proposed in this paper.

#### 2. Surface temperature variations

With the formation of an embryo bubble, the change in the Gibbs function is expressed as [1]

$$\Delta G = \frac{4}{3}\pi r^3 \rho_{\rm v} (G_{\rm v} - G_{\rm l}) + 4\pi r^2 \sigma \tag{1}$$

#### Nomenclature

where  $G_v$  and  $G_l$  are the Gibbs free energies for the vapor and the liquid phase.  $\sigma$  is the surface tension,  $\rho_v$  is the vapor density, and *r* denotes the bubble radius.

Assuming that the bubble is in equilibrium with the surrounding liquid, the pressure difference across the interface satisfies the Young–Laplace equation, i.e.

$$p_{\rm v} - p_{\rm l} = \frac{2\sigma}{r} \tag{2}$$

The pressure increase in the bubble is related to the increased saturation temperature by the well-known Clausius–Clapeyron equation as

$$\frac{\Delta T_{\rm s}}{\Delta p} = \frac{T_{\rm s}}{h_{\rm fg}\rho_{\rm v}} \tag{3}$$

Substituting Eq. (2) into Eq. (3), yields

$$\Delta T_{\rm s} = \frac{2\sigma T_{\rm s}}{h_{\rm fg}\rho_{\rm v}r} \tag{4}$$

The equilibrium bubble radius can then be written as

$$r_{\rm e} = \frac{2\sigma T_{\rm s}}{\Delta T_{\rm s} h_{\rm fg} \rho_{\rm v}} \tag{5}$$

Setting the derivative of Eq. (1) with respect to r equal to zero, yields

$$r_{\rm c} = -\frac{2\sigma}{(G_{\rm v} - G_{\rm l})\rho_{\rm v}}\tag{6}$$

where  $r_c$  is the critical radius of a bubble. For the instant,  $t_c$ ,  $r_e = r_c$ , Eqs. (5) and (6) can be compared to give

$$G_{\rm v} - G_{\rm l} = -\frac{h_{\rm fg}\Delta T_{\rm s}}{T_{\rm s}} \tag{7}$$

Substituting Eq. (7) into Eq. (1), yields

$$\Delta G = -\frac{4\pi\rho_{\rm v}h_{\rm fg}\Delta T_{\rm s}}{2T_{\rm s}}r^3 + 4\pi\sigma r^2 \tag{8}$$

For a given system,  $\rho_{\rm v}$ ,  $h_{\rm fg}$ ,  $\Delta T_{\rm s}$ ,  $T_{\rm s}$ , and  $\sigma$  are known constants,  $\Delta G$  will be a function of the single variable r, and hence, the following relationship would be satisfied

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{\partial\Delta G}{\partial r} = \frac{4\pi\rho_{\rm v}h_{\rm fg}\Delta T_{\rm s}}{T_{\rm s}}r^2 - 8\pi\sigma r \tag{9}$$

The nonlinear effects involved in pool boiling systems include the non-uniform site distribution, the on/off behavior of sites, the formation and evaporation of microlayers and macrolayers, the bubble generation, growth, departure and coalescence, and the interactions between bubbles or sites. All these nonlinear effects are concentrated in the boundary layer near the heater surface. Therefore, the nonlinear behavior of pool boiling systems can be investigated by solving the energy conservation equation and the bubble dynamic equation in a control volume of the two-phase system in the boundary layer adjacent to the heater surface.

In the nucleation process, the surface temperature is extremely non-uniform and rapidly varies with time because of the nonlinear interactions between active sites or bubbles. Assuming that, the parameters of the two-phase system near the surface are homogenized throughout the whole volume and the thermophysical properties are constant, the temperature of heating wall can be determined by the heat balance for the control volume of the two-phase system as

$$\rho c \frac{\mathrm{d}T}{\mathrm{d}t} = (\alpha - \alpha_{\rm c})(T - T_{\rm s}) - \rho_{\rm v} h_{\rm fg} \int_0^\infty \frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{4}{3}\pi r^3\right) N(r,\xi) \,\mathrm{d}r$$
(10a)

where *T* denotes the surface temperature, or  $\Delta T = (T-T_s)$  is the wall surface temperature superheat, and  $\alpha$  is the overall heat transfer coefficient which can be assumed to be exponentially related to the average superheat. The convection heat transfer coefficient,  $\alpha_c$ , including bulk convection and natural convection is that proposed by Jaoyhuob [9].  $N(r, \xi)$  is the bubble size distribution function proposed in the following sections. The bubble size is evaluated from Eq. (9), which is rewritten as

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{4\pi\rho_{\rm v}h_{\rm fg}(T-T_{\rm s})}{T_{\rm s}}r^2 - 8\pi\sigma r \tag{10b}$$

### 3. Bubble number model

The boiling process is a non-equilibrium, irreversible kinetic process whose nature is determined by embryo (or bubble) nucleation, growth and movement under the action of superheat or heat fluxes. Boiling may be principally divided into two stages, the nucleation and growth of a single embryo or bubble, and the spreading of dry patches formed by bubble coalescence. The boiling process is stochastic, but not deterministic. Hence, the whole process is also statistical, which relates the microscopic mechanisms to macroscopic effects.

The number of nucleation sites is distributed randomly over a boiling surface. Judd and Chopra [8] demonstrated that a clustering effect exists since the bubble flux density was non-uniformly distributed over the boiling surface. Clustering suggests the existence of interactions among active sites and bubbles, and hence, the bubble growth is stochastic such that the bubble radius, r, may be modeled as an average background with superimposed inhomogeneous fluctuations. The effects of interactions between bubbles can be, therefore, included in the stochastic behavior of boiling system.

Let t denote the time for which a wall is subjected to the applied heat fluxes, then  $\dot{r} = (dr/dt)$  is the bubble growth rate which must obey the following generalized Langevin equation [11]

$$\dot{r} = k(r) + F(t) \tag{11}$$

where k(r) is the drift growth rate determined by nonsystem conditions and F(t) is the fluctuation growth rate determined by the inhomogeneous fluctuations. In general, the bubble growth process may be approximately regarded as a Markov process and the fluctuation F(t) may be assumed to have a Gaussian distribution for convenience, i.e.

$$\langle F(t)F(t')\rangle = Q\delta(t-t') \tag{12}$$

Here  $\delta$  is the Dirac delta function and Q is the fluctuation growth coefficient.

From stochastic theory, the generalized Fokker– Planck equation [11], which corresponds to the generalized Langevin Eq. (11) and Eq. (12), is as follows

$$\frac{\partial P(r,t)}{\partial t} = -\frac{\partial}{\partial r} [k(r)P] + \frac{Q}{2} \frac{\partial^2 P}{\partial r^2}$$
(13)

Eq. (13) is the differential equation describing the stochastic growth process of an embryo bubble, where P(r,t) is the probability of bubbles between r and (r + dr) at time t, and satisfies the normalization condition as

$$\int_0^\infty P(r,t) \,\mathrm{d}r = 1 \tag{14}$$

A real boiling system includes not just a few embryos or bubbles nucleating and growing, but a large number of embryos or bubbles nucleating and growing simultaneously due to the applied heat flux. The differential equation describing the number of embryos or bubbles growing in the system can then be related to the probability.

The dimensionless parameter

$$\xi = \frac{\rho_{\rm v} h_{\rm fg}}{\rho_{\rm l} C_{\rm pl} \Delta T} \tag{15}$$

which is the inverse of the Jacob number defined as  $Ja = (\rho_1 C_{pl} \Delta T)/(\rho_v h_{fg})$ , is an important parameter controlling the boiling process.  $N(r,\xi)$  dr is the average bubble number having radius r at  $\xi$ . The average bubble number having radius between r and (r + dr) at  $(\xi - d\xi)$  is

$$N(r, \xi - d\xi) dr = \left(N - \frac{\partial N}{\partial \xi} d\xi\right) dr$$
(16)

The net increase in bubble number from  $\xi$  to  $(\xi - d\xi)$  would be

$$-\frac{\partial N}{\partial \xi} \,\mathrm{d}\xi \,\mathrm{d}r \tag{17}$$

The bubble number increase comes from two mechanisms: bubble growth and nucleation. The average bubble number growing from r to (r + dr) from  $\xi$  to  $(\xi-d\xi)$  is  $[k(r)N]_r$ , where k(r) is the drift bubble growth rate. The average bubble number with radii exceeding (r + dr) and with initial radii between r and (r + dr) is

$$[k(r)N]_{r+\mathrm{d}r}\,\mathrm{d}\xi = \left[(k(r)N)_{\mathrm{r}} + \frac{\partial}{\partial r}(k(r)N)_{\mathrm{r}}\,\mathrm{d}r\right]\mathrm{d}\xi \qquad (18)$$

So, the net increase in the average bubble number between r and (r + dr) due to growth will be

$$-\frac{\partial}{\partial r}[k(r)N] \,\mathrm{d}r \,\mathrm{d}\xi \tag{19}$$

The average number of the newly generated bubble between *r* and (r + dr) for  $\xi$  to  $(\xi-d\xi)$  is

$$q(r,\xi) \,\mathrm{d}r \,\mathrm{d}\xi \tag{20}$$

where  $q(r, \xi)$  dr is the average bubble number per unit area between r and (r + dr) at  $\xi$  due to nucleation. Combining Eqs. (17), (19) and (20), yields

$$-\frac{\partial N(r,\xi)}{\partial \xi} = q(r,\xi) - \frac{\partial}{\partial r} [k(r)N(r,\xi)]$$
(21)

Assuming  $q(r, \xi)$  to be expressed by

$$q(r,\xi) = q(\xi)\delta(r) \tag{22}$$

where  $\delta$  is the Dirac delta function, then Eq. (21) can be rewritten as

$$-\frac{\partial N}{\partial \xi} = q(\xi)\delta(r) - \frac{\partial}{\partial r}[k(r)N]$$
(23)

Here, the first term is the increase due to nucleation and the second term is the increase due to growth. Eq. (23) satisfies the initial and boundary conditions

$$N(r, \xi \to \infty) = 0 \tag{24}$$

$$N(r \to \infty, \xi) = 0 \tag{25}$$

Eq. (22) can be solved to determine  $N(r, \xi)$  for a given  $q(\xi)$  and  $k(r, \xi)$ . The probability for bubbles between r and (r + dr) is then determined from



Fig. 1. Embryo radius for 1 K average superheat.

$$P(r,\xi) dr = N(r,\xi) dr/N(\xi)$$
(26)

where  $N(\xi) = \int_0^\infty N(r, \xi) dr$  is the total bubble density.

The differential equations for bubble evolution can be derived, by combining Eqs. (23)–(25), as

$$-\frac{\partial N(\xi, r)}{\partial \xi} + \frac{\partial}{\partial r} [\overline{k(r)} N(\xi, r)] = 0$$
(27)

with corresponding boundary conditions:

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$$N(\xi \to \infty, r) = 0 \tag{28a}$$

$$N(\xi, r \to \infty) = 0 \tag{28b}$$

$$[k(r)N]_{r=0} = q(\xi)$$
(28c)

Benjamin and Balakrishnan [12] have developed a correlation for the nucleation site density in terms of the



Fig. 2. Attractor for 1 K average superheat.



Fig. 3. Bubble radius for 7 K average superheat.

thermophysical properties of heating plate and liquid and metrological properties of surface, which fits a large amount of experimental data and is employed for  $q(\xi)$ 

$$q(\xi) = 218.8(Pr)^{1.63} \left(\frac{1}{\gamma}\right) \theta^{-0.4} \left(\frac{\rho_v h_{\rm fg}}{\rho_{\rm l} C_{\rm pl}}\right)^3 \xi^{-3}$$
(29)

where

$$\theta = 14.5 - 4.5 \left(\frac{R_a p}{\sigma}\right) + 0.4 \left(\frac{R_a p}{\sigma}\right)^2 \tag{30a}$$

$$\gamma = \sqrt{\frac{\lambda_{\rm w}}{\lambda_{\rm l}}} \frac{\rho_{\rm w} C_{\rm pw}}{\rho_{\rm l} C_{\rm pl}} \tag{30b}$$

Pr is the Prandtl number defined as  $Pr = v\rho C_p/\lambda$ , p is the system pressure and  $R_a$  is the surface roughness.



Fig. 4. Attractor for 7 K average superheat (+7' denotes *y*-coordinate moved to left for 7 K).



Fig. 5. Bubble radius for 15 K average superheat.

There are many correlations describing the bubble growth rate. If the heat is transferred to the bubble from the liquid microlayer underneath the bubble in the form of latent heat, the average bubble growth rate was determined by Labuntsov [13] as

$$\overline{k(r,\xi)} = \hat{c} \frac{\lambda_{\rm l}}{\rho_{\rm l} C_{\rm pl}} \xi^{-1} \tag{31}$$

with constant  $\hat{c} = 5$  to 10. Solving Eq. (27) yields

 $\hat{c}\lambda_1$ 

$$N(\xi, r) = \frac{218.8(Pr)^{1.03} \left(\frac{1}{\gamma}\right) \theta^{-0.4} \left(\frac{\rho_v h_{\rm fg}}{\rho_1 C_{\rm pl}}\right)^3 C_{\rm pl} \rho_1}{\hat{c} \lambda_1 \xi^2}$$
$$\exp\left(-\frac{2\rho_1 C_{\rm pl}}{\hat{c}}r\right) \tag{32}$$



Fig. 6. Attractor for 15 K average superheat (+10' denotes *y*-coordinate moved to left for 10 K).

Eq. (32) means that, the bubble nucleation site density will be inversely related to the square of  $\xi$  and decreases exponentially with increasing bubble radius.

The bubble probability can be obtained from Eq. (26) as

$$P(\xi, r) = \frac{2\rho_1 C_{\rm pl}}{\hat{c}\lambda_1} \exp\left(-\frac{2\rho_1 C_{\rm pl}}{\hat{c}\lambda_1}r\right)$$
(33)

Therefore, the bubble probability decreases exponentially with increasing bubble radius.

#### 4. Pool nucleate boiling heat transfer

We consider the most important contribution to the nucleate boiling heat transfer belongs to latent heat and convection mainly including micro-convection and natural convection heat transfer associated with the nucleate boiling process. Accounting for the nonlinear aspects of the surface temperature variation and bubble interaction, we propose a new method to predict the nucleate boiling heat transfer.

If the latent heat transported from the surface by the bubbles departing from a single nucleation site is  $q_e''(r)$ , the overall latent heat transfer  $q_e''$  can be calculated as

$$q_e'' = \int_0^\infty q_e''(r) N(r, \xi) \, \mathrm{d}r$$
  
$$= \rho_v h_{\mathrm{fg}} \int_0^\infty \frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{4}{3}\pi r^3\right) N(r, \xi) \, \mathrm{d}r$$
 (34)

Using Eq. (32), gives

$$q_{e}'' = \frac{218.8\pi (p_{\rm r})^{1.63} \theta^{0.4} \left(\frac{\rho_{\rm v} h_{\rm fg}}{\rho_{\rm l} C_{\rm pl}}\right)^4 \hat{c} \lambda_1 a_{\rm l}}{2\gamma \xi^2}$$
(35)  
$$(0.3\xi^{-1} + \sqrt{0.09\xi^{-2} + 12\xi^{-1}})$$

Convective heat transfer in nucleate boiling mainly includes: (a) the micro-convection heat transfer when the liquid layer replacing departing bubbles contacts the heating surface. (b) The natural convection heat transfer (or Marangoni flow heat transfer which may be the same order of magnitude) along the surface not occupied by bubbles. Jaoyhuob [9] proposed a coupled convection and evaporative heat transfer model for fully-developed nucleate pool boiling,

$$q_{\rm tot}'' = q_{\rm e}'' + q_{\rm c}'' \tag{36}$$

with the convection heat transfer flux  $q^{\prime\prime}_c$  can be calculated as



Fig. 7. Comparison of the prediction with experimental data [14] (copper plate, roughness  $R_a = 0.07 \mu m$ ): (a) for water; (b) for CCl<sub>4</sub>; (c) for acetone; (d) for *n*-hexane.

$$q_{\rm c}'' = 10^{-3} \times \frac{\lambda_1^2 (\Delta T)^3}{\sigma T_{\rm s} v_1}$$
(37)

We adapt Eqs. (35)–(37) thereby to predict the total heat transfer flux for pool nucleate boiling.

## 5. Predicting results with discussions

## 5.1. Nonlinear aspects of the surface temperature

Eqs. (10a) and (10b) were used to investigate pool boiling of water at atmospheric pressure, using time



Fig. 8. Comparison of the prediction with experimental data [15] (nickel plate, roughness  $R_a = 0.045 \mu m$ , for *n*-pentane).

step of 0.001 s and 3600 iterations per time step, for  $\alpha = 2\Delta T_{\text{ave}}^{1.5}$ ,  $\alpha_{\text{c}} = 0.5\Delta T_{\text{ave}}^2$ , and  $T_{\text{s}} = 100^{\circ}\text{C}$ . The bubble size distribution function is given by Eq. (32) we derived.  $\Delta T_{ave}$  is the average superheat, which is the difference between the average surface temperature  $T_{\rm ave}$  and liquid saturated temperature  $T_{\rm s}$ .  $T_{\rm ave}$  can be obtained by numerical integration. Typical bubble sizes and attractor at different average superheats are shown in Figs. 1-6. Wave characteristics in Figs. 1, 3 and 5 indicate that the bubble size varies with time. The unit of x coordinate is millisecond. The attractor represents the relationship between the bubble size and the average superheat. At small average superheats, the bubble sizes damp with time and the attractor is a fixed dot. When the average superheat reaches a certain value, the bubble size varies cyclically with time and the attractor has the form of a circle. When the average superheat reaches a higher value, the bubble size varies chaotically with time indicating that for high heat flux, the systems must be modeled using theory of chaos. The values where bifurcation and chaos occur depend on the specified conditions. The most important thing is not the absolute values, but the existence of chaos in the boiling system. The active nucleation site density will change with time even for a given average superheat or heat flux.

Current physical models based on a single site or a single vapor bubble, assuming uniform conditions and prescribed site density distributions, do not consider possible changes of the active site density. Therefore, it would be expected that, the existing models are inconsistent with most experimental data for nucleate boiling in the literature. Future efforts must address the nonlinear interactions and the chaotic features of the physics to correlate experimental data or to accurately predict the heat transfer [10]. 5.2. Comparison of the present model with experimental results

Using the new method described by Eqs. (35)–(37), the nucleate boiling heat transfer was calculated corresponding to the experimental conditions in several investigations as reported in [2,14,15]. The results are presented in Figs. 7–9. The predictions by the present model are also compared with the experimental data of Kurihara and Myers [14], Zuber [15] and Benjamin and Balakrishnan [2].

The results in Figs. 7 and 9 correspond to four different boiling liquids. The heating surface in Fig. 7 is a copper plate with roughness  $R_a = 0.07 \ \mu m$ , while that in Fig. 9 is an aluminum plate with roughness  $R_{\rm a} = 0.52 \ \mu {\rm m}$ . Fig. 8 illustrates the prediction and comparison with the measurement data by Zuber [15] for nickel plate with roughness  $R_a = 0.045 \ \mu m$  and liquid n-pentane. Apparently, at the same superheat, the heat flux for the copper plate is higher than that for the aluminum plate. This means that the bubble interaction and surface temperature nonlinear aspect have less influence in decreasing the heat transfer, since the thermal conductivity of copper is much better than that of aluminum. Actually, for copper plate, the bubble interaction and nonlinear aspect result in less non-uniformity of surface temperature than for aluminum. As shown in Figs. 7 and 9, the present model is in good agreement with experimental data.

The data shown in Figs. 7–9 also show that, for a same superheat, the heat flux for water is almost one order of magnitude greater than that of organic liquids. This may be attributed to the fact that the latent heat of water is almost one order of magnitude greater than that of organic liquids.

Based on the survey of the available models and physical understanding of nucleate pool boiling, an analytical model is proposed here to predict nucleate boiling heat transfer, the contribution due to evaporative heat transfer is a predominant factor, which differs from Mikic and Rohsenow's model [6], but is similar to that of Judd and Huang [8]. The present model provides an improved understanding of the mechanism of nucleate pool boiling. Its application requires only the parameters describing the heating surface features and the physical properties.

#### 6. Conclusions

Available empirical or phenomenological correlations and physical models were developed using averaged linear approximations. They can be applied only in relatively narrow ranges with considerable uncertainty. The transitions between boiling modes should



Fig. 9. Comparison of the prediction with experimental data [2] (aluminum plate, roughness  $R_a = 0.52 \mu m$ ): (a) for water; (b) for CCl<sub>4</sub>; (c) for acetone; (d) for *n*-hexane.

be related to the formation and spreading process of embryos, bubbles and dry patches.

The present investigation considers the interactions among active sites or bubbles and the nonlinear dynamics of the interactions, and analyzes the stochastic and nonlinear features of the boiling systems. A bubble-size statistical distribution function has been derived from a bubble number balance to develop a new method for predicting the nucleate boiling heat transfer. The results are in good agreement with available experimental data in the literature. Hence, the suggested method is more reasonable for analyzing pool nucleate boiling heat transfer research incorporating the nonlinear and stochastic effects. Further studies are needed to validate this approach for flow nucleate boiling.

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